A Solution to Problem #2

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In Lab Activity #2, "The Seasons", we used satellite observations to calculate the area-weighted, global average insolation, averaged over the month of December, 1987: 361.8 W/m^2. For June we calculated a value of 340.0 W/m^2. Why do these differ? One possibility is that the sun was brighter in December than in June of 1987. Another is that the earth is closer to the sun in December than in June, and solar intensity decreases with increasing distance from the sun. Both explanations are qualitatively consistent with the observations (hence, we can't rule them out), though there might be other possible explanations as well.

To test the varying distance from the sun explanation further, we will try to calculate what the global-average insolation should be at those two months based partly on theory, and compare the theoretical results to the observed ones. If they are close, this will support (but still not prove) the distance-from-the-sun hypothesis as an explanation for the different global average insolation values in December and June.

We start with the theoretical calculation, using the "Good Problem Solving Strategy" format.

I. Information Given or Otherwise Known

- Maximum distance between earth and sun (at aphelion)
  $\equiv (r_{e2s})_{\max} = 1.521\times10^8$ km
- Minimum distance between earth and sun (at perihelion)
  $\equiv (r_{e2s})_{\min} = 1.471\times10^8$ km
- Average distance between earth and sun $\equiv \overline{r}_{e2s} = 1.496\times10^8$ km
- Solar radiative energy flux directly facing the sun at the earth's average distance from the sun (that is, the solar constant)
  $\equiv S(\overline{r}_{e2s}) = 1370$ W/m^2

II. Information Desired

- Calculated global average solar radiative energy flux for December $\equiv S\left(r_{e2s}(Dec)\right)$
- Calculated global average solar radiative energy flux for June
  $\equiv S\left(r_{e2s}(Jun)\right)$

III. Relations Needed

(1) $\overline{S} = SR/A_{sfc}$

[Relation between global average solar radiation flux and total rate at which solar radiation strikes the earth, where:

$\overline{S} \equiv$ global average solar radiative energy flux

$SR \equiv$ total rate at which solar radiation strikes the earth

$A_{sfc} \equiv$ surface area of the earth]

(2) $SR = S(r_{e2s}) \times A_{X\text{-}sect}$

[Relation between total rate at which solar radiation strikes the earth and the cross-sectional area of the earth facing the sun, where:

$r_{e2s} \equiv$ distance between the earth and the sun

$S(r_{e2s}) \equiv$ flux of solar radiation on a surface facing the sun, at the distance between the earth and sun

$A_{X\text{-}sect} \equiv$ cross-sectional area of the earth directly facing the sun]

(3) $A_{X\text{-}sect} = \pi R_e^2$

[Relation between cross-sectional area of the earth (a circle) and the earth's radius, where:

$R_e \equiv$ radius of the earth]

(4) $A_{sfc} = 4\pi R_e^2$

[Relation between surface area of a the earth (a sphere) and the earth's radius.]

(5) $S(r_{e2s}) = S(\overline{r}_{e2s}) \times \left[\frac{\overline{r}_{e2s}}{r_{e2s}}\right]^2$

[Inverse square law applied at the distance between earth and sun. Note that this produces the same results if the solar radiative flux at any other known distance from the sun replaces $S(\overline{r}_{e2s})$ and $\overline{r}_{e2s}$. An example would be the solar radiative flux at the surface of the sun, which is just the solar radiative emission flux.]

IV. Solution

(A) Substitute Eq. (2) into Eq. (1):

$$\text{(6)}\quad \overline{S} = S(r_{e2s}) \times A_{X\text{-}sect} \big/ A_{sfc}$$

(B) Substitute Eqs. (3) and (4) into Eq. (6):

$$\text{(7)}\quad \begin{aligned} \overline{S} &= SF(r_{e2s}) \times \left(\pi R_e^2\right) \Big/ \left(4\pi R_e^2\right) \\ &= S(r_{e2s})/4 \end{aligned}$$

(C) Substitute Eq. (5) into Eq. (7):

$$\text{(8)}\quad \overline{S} = \frac{S(\overline{r}_{e2s})}{4} \times \left[\frac{\overline{r}_{e2s}}{r_{e2s}}\right]^2$$

(D) Apply Eq. (8) to the minimum and maximum distances from the sun.

We don't know the average distance between the earth and sun in December or June, but the earth is closest to the sun in early January and farthest in early July, which are pretty close to December and June, respectively. Hence, let's assume that the global average solar fluxes at the earth's closest and farthest points from the sun can approximate the global average solar fluxes in December and June, respectively:

$$\text{(8)(a)}\quad \overline{S}\left(r_{e2s}(Dec)\right) \cong \overline{S}\left(\left(r_{e2s}\right)_{min}\right) = \frac{S(\overline{r}_{e2s})}{4} \times \left[\frac{\overline{r}_{e2s}}{\left(r_{e2s}\right)_{min}}\right]^2$$

$$\text{(8)(b)}\quad \overline{S}\left(r_{e2s}(Jun)\right) \cong \overline{S}\left(\left(r_{e2s}\right)_{max}\right) = \frac{S(\overline{r}_{e2s})}{4} \times \left[\frac{\overline{r}_{e2s}}{\left(r_{e2s}\right)_{max}}\right]^2$$

(E) Substitute values and units into Eqs. 8(a) and (b):

$$\bar{S}(r_{e2s}(Dec)) \cong \frac{1370\ W/m^2}{4} \times \left[\frac{1.496\times10^8 km}{1.471\times10^8 km}\right]^2$$

$$= \frac{1370}{4}\left[\frac{1.496}{1.471}\right]^2 \times \left[\frac{10^8}{10^8}\right]^2 \frac{W}{m^2}\left[\frac{km}{km}\right]^2$$

$$\boxed{\bar{S}\left(r_{e2s}\left(Dec\right)\right) \cong 354.2W / m^2}$$

$$\bar{S}(r_{e2s}(Jun)) \cong \frac{1370\ W/m^2}{4} \times \left[\frac{1.496\times10^8 km}{1.521\times10^8 km}\right]^2$$

$$= \frac{1370}{4}\left[\frac{1.496}{1.521}\right]^2 \times \left[\frac{10^8}{10^8}\right]^2 \frac{W}{m^2}\left[\frac{km}{km}\right]^2$$

$$\boxed{\bar{S}\left(r_{e2s}\left(Jun\right)\right) \cong 331.3W/m^2}$$

V. Check Solution

Both values are positive, as we expect, and have units of energy flux, as we want. The value for December, when the earth is closer to the sun, is larger than it is for June, when the earth is farther from the sun, as we expect.

Comparison with Fluxes Calculated from Observations

The values that we calculated from theory, above—354.2 W/m^2 and 331.3 W/m^2, respectively—are both lower than the observed values, by 7.6 W/m^2 for December and 8.7 W/m^2 for June (about 2.1% and 2.5%, respectively). The two calculated values differ from each other by 22.9 W/m^2 (about 6.7%), which is very similar to the amount by which the observed values differ from each other (361.8 W/m^2 – 340.0 W/m^2 = 21.8 W/m2, about 6.2%), especially if we take into account the fact that the difference between the December and June average insolation should be a little smaller than the difference between the global-average insolation values at the solstices, which is what we actually calculated.

There are several sources of error here that might account for the difference between the theoretical and observed results. For example:

(1) The value used here for the solar constant (1370 W/m^2) might differ a bit from the value cited by other sources or calculated using the emission flux of the sun using the Stefan-Boltzmann Law.

(2) The earth is not quite as close to the sun in December as it is at its closest approach in early January, so we might expect the value that we calculated here might be a slight overestimate, but in fact it is lower than observed for December, by 7.6 W/m^2. Also, the earth is not quite as far from the sun in June as it is at its farthest point in early July, so we might expect the value calculated here to be a slight underestimate, and we calculate a value 8.7 W/m^2 higher than observed for June, so the difference is at least in the right direction.

(3) In our calculations, we might use minimum and maximum distances between the earth and the sun that have a little more precision than the values that we actually used.

(4) The observations themselves, or our calculation of the area-weighted global average of the observations, might have errors in them.

Without further investigation, we don't know if our results would be better or worse if we made adjustments to reduce these sources of possible error, but in some respects, at least (notably the difference between the two calculated values and between the two observed values, which are very similar), our theoretical results are close enough to provide rather strong quantitative support for the proposed explanation that observed variations in global average insolation vary with time of year due to variations in distance between the earth and sun.

Addendum: Upon further investigation, using a different file with what should be the same monthly-average insolation data (in the Planetary Radiation Budget project that we used in Lab #4, rather than the Monthly Average Incoming Solar Radiation project that we used in Lab #2), we can use My World to find an area-weighted global and monthly average insolation in December of 352.1 W/m^2 and in June of 330.9 W/m^2. The values that we calculate from theory at the solstices above differ from these alternative observed values by only 2.1 W/m^2 (albeit still higher than the alternative observed value, contrary to what we would expect) and 0.4 W/m^2 (which is still a bit lower than the alternative observed value, as we would expect), respectively. Like area-weighted values that we calculated using the Monthly Average Incoming Solar Radiation project data, these alternative observed values differ from each other by 6.2%, a bit less than the calculated values, as we would expect.

Hence, we find an inconsistency between what should be duplicate versions data sources. This sort of thing can always happen, and it is one reason why we can never be entirely certain about the results of scientific reasoning and investigations.