# Heat Capacity and Specific Heat

Consider a chunk of matter, which might be solid or might be a "parcel" of water or gas. Either way, we'll call it an "object".

The ***heat capacity*** of the object is a measure of how much heat the object must gain or lose to change its temperature by a given amount. In the MKS (meters/kilograms/seconds) system of measurement, heat capacity would be expressed in units of Joules per degree centigrade (°C)—that is, the heat capacity of the object would be the amount of heat (in Joules) that the object would have to gain or lose for its temperature to change by 1°C. Another common unit of heat capacity is the calorie per °C, where one calorie is defined as the amount of heat required to raise the temperature of 1 gram of pure water (at 3.98°C, 14.5°C, or 19.5°C, depending on who's doing the defining) by 1°C at standard sea level pressure.

What determines an object's heat capacity? One obvious answer is the *amount of matter in the object* (expressed in terms of its mass). The more "stuff" there is in the object—that is, the greater its mass—the more heat it would have to gain or lose for its temperature to change by 1°C. For example, it would take a lot more heat to warm the Pacific Ocean by 1°C than it would take to warm a glass of seawater by 1°C!

The other answer is that the heat capacity depends on the *type of material of which the object is composed*—not all materials require the same amount of heat gain or loss to change temperature by 1°C. That is, to change temperature by 1°C , objects made of some materials have to gain or lose more heat than do objects made of other materials, even when the objects have the same mass. For example, liquid water turns out to be one of the hardest substances found in nature to warm up or cool off—it requires a greater gain or loss of heat to change its temperature by 1°C than the same mass of most natural substances on earth.

Heat capacity might depend on other things, too, such as the temperature of the object or the atmospheric pressure. For a gas, heat capacity would depend on whether pressure were being held constant during the heat gain or loss, or whether the volume were held constant, or neither.

A relation between the heat gained or lost ($\Delta H$) by an object and the change in temperature it undergoes can be written:

$$\Delta H = C_H \Delta T$$

where $C_H$ is the object's heat capacity (which depends on the various things described above).

The ***specific heat*** of a material is related to heat capacity, except that specific heat doesn't depend on an object's mass, though it still depends on the type of material. Specific heat is therefore a *property of the material*.

Specific heat is defined as the amount of heat that a *unit* mass of a material must gain or lose to change its temperature by a given amount. In MKS units it would be expressed in terms of Joules per kilogram per °C. A common alternative unit is calories per gram per °C. As implied above, by definition of the calorie the specific heat of pure water is 1 cal/gm/°C (at 3.98°C, 14.5°C, or 19.5°C, depending on who's doing the defining) at standard sea level pressure.

The specific heat of a material is related to the heat capacity, $C_H$, of an object made of that material, as follows:

$$c_H = C_H / m$$

where $c_H$ is the specific heat and $m$ is the mass of the object. Substituting into the earlier relation above, we derive a relation between the heat gained or lost by an object, the object's mass, its specific heat, and its change in temperature:

$$\Delta H = c_H m \Delta T$$

We will take advantage of this relation to estimate the specific heat of sand, which we will take to be broadly representative of the specific heat of "land" on earth.

**Measuring the Specific Heat of Sand**

Given that the specific heat of water is 1 calorie/gm/°C (which we will take to be representative of sea water and hence "ocean"), we can measure the specific heat of sand (which we will take as representative of "land") using the experimental procedure described below.

**Materials Needed**

-- thermos or other insulated container     --beaker
-- thermometer     -- weighing scale
-- water     -- hot plate
-- sand

**Procedure**

(1) Record the temperature of the room in degrees centigrade. (This should also be the initial temperature of the sand, $T_s(t_0)$, assuming that the sand has been sitting around in the room sufficiently long.)

(2) Place the thermos on the scale and record its mass in grams.

(3) Add water to the beaker (no more than half full), place the beaker on the hot plate, and heat the water to a temperature lower than the maximum temperature measurable by the thermometer.

(4) Pour heated water into the thermos (no more than half full) and record its mass. Subtract the result of (2) from the result of (4) to determine the mass of water in the thermos, $m_w$.

(5) Record the temperature of the water, $T_w(t_0)$, when the temperature has largely stabilized (or is changing only very slowly).

(6) Add sand to the water in the thermos (less than one quarter of the total volume of the thermos). Record the mass of the partly filled thermos and subtract the mass measured in (4) to determine the mass of sand, $m_s$, added to the thermos.

(7) Cover the top and swish the sand around in the water briefly. Repeatedly record the temperature of the water/sand mixture and swish it around, until the temperature has largely stabilized. We'll call this the "final" temperature, $T(t_f)$. It should be the temperature of both the sand and the water.

**The Calculation**

The calculation of the specific heat of the sand hinges on several assumptions, some better than others:

- When the approximately room-temperature sand is added to the heated water, the sand will warm and the water will cool by conduction, and the amount of heat lost by the water will equal the heat gained by the sand (that is, heat will be conserved). For simplicity, assume that no significant amount of heat is lost by the heated water to the thermos, the thermometer, or the air (by conduction or evaporation).

- The sand and water end up at the same temperature.
- The sand consists of material that all has the same specific heat.

The relations between temperature change and heat lost by the water and gained by the sand can be written separately as:

(1)
$$\Delta H_w = c_{Hw} m_w \Delta T_w \equiv c_{Hw} m_w \left[ T_w(t_f) - T_w(t_0) \right]$$
$$\Delta H_s = c_{Hs} m_s \Delta T_s \equiv c_{Hs} m_s \left[ T_s(t_f) - T_s(t_0) \right]$$

where $c_{Hs}$ and $c_{Hs}$ are the specific heats of water and sand, respectively. By assumption, the heat gained by the sand equals the heat lost by the water:

(2) $$\Delta H_s = -\Delta H_w$$

Substituting Eqs.(1) into Eq.(2) to eliminate the heat gain/loss terms gives:

(3) $$c_{Hs} m_s \left[ T_s(t_f) - T_s(t_0) \right] = -c_{Hw} m_w \left[ T_w(t_f) - T_w(t_0) \right]$$

The assumption that the sand and water end up at the same temperature can be written as:

(4) $$T_f \equiv T_w(t_f) = T_s(t_f)$$

Substituting Eq.(4) into Eq.(3) gives:

(5) $$c_{Hs} m_s \left[ T_f - T_s(t_0) \right] = -c_{Hw} m_w \left[ T_f - T_w(t_0) \right] = c_{Hw} m_w \left[ T_w(t_0) - T_f \right]$$

We have measured or otherwise know everything in Eq.(5) except $c_{Hs}$, so we can solve for it:

(6) $$c_{Hs} = c_{Hw} \left( \frac{m_w}{m_s} \right) \frac{\left[ T_w(t_0) - T_f \right]}{\left[ T_f - T_s(t_0) \right]}$$

So, based on your measurements, how does the specific heat of "land" compare to the specific heat of "ocean"?

(The actual specific heat of quartz sand is 0.19 cal/gm/°C, though for sandy clay, which is more like soil, it's 0.33 cal/gm/°C.)